**Excitations**

I’m going to retread our results a little, coming from a different direction – using Green’s functions.

**Singlet-singlet correlation function**

Still using BCS model. We can kind of do the same thing in a different way. So in this section we will show that the region of attractive effective electron-electron interactions leads at sufficiently low temperatures to an instability in the homogeneous electron gas towards another state which turns out to be a superconducting state. Let’s go back to:



where,



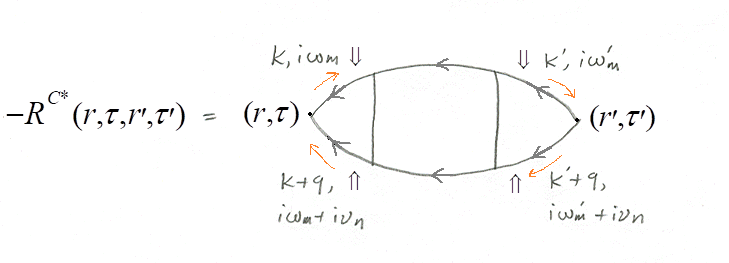
and calculate the singlet-singlet density wave correlation function. The singlet ψ↑†(r,t)ψ↓†(r´,t’) is closely related to the Cooper pair ck↑†(t)c-k↓†(t’). So we want,



It measures something like the likelihood that a singlet formed at (r’,t’) will still be around at (r,t). This would examine the ‘eigenstateness’ of singlet density waves, so we should expect to ascertain the energy spectrum of these singlet states by examining the GF pole structure. This calculation is nice because we’re not doing the whole multi-particle overlap states thing. Let us calculate this correlation function (the pair susceptibility) via:



It looks like this:



and we can write it as:



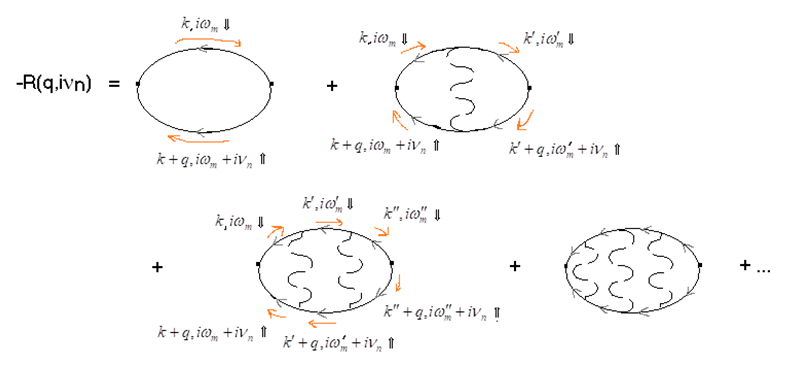
where […..] is the rest of the diagram translated into Fourier space [see that FT file, and the metal/impurities+electrons/non-equilibrium/conductivity/quantum file too for similar analysis]. Proceeding,



Then taking the FT,



The 1/V2 and 1/β2 is just the usual factor associated with any sum over internal momentum/energy transfers. So the Feynman rules are just the usual, except additionally there will be implicit θ functions on each GF0 restricting its momentum to the ωD interval about the Fermi surface. The leading diagrams contributing to RC\*(q,τ) are ladder diagrams where the rungs are given by the model e-e interaction.



This works out to:



which we can write as:



Then if we define,



we may write,



and so finally,



We’ve calculated Λ, or something like it, before. It’s similar to Π0(RPA)C\*(q,iνn), which we evaluated in the Interacting Electrons Collective Oscillations file. So let’s do that real fast:



Note,



So,



Let us now analytically continue to the real axis and take |q| → 0, ω → 0 (which maximizes Λ). So now we’re examining the singlet density wave which has an infinitely long wavelength, and no oscillation frequency. So we’re looking at the state where basically everywhere, we have a static singlet state. Now since ξ-k = ξk.



And observe,



Now we’ll convert to an integral, finally explicitly taking account that the momentum is restricted to the ωD interval about the Fermi surface. We’ll also presume the density of states to be constant within that interval. So,



We’re interested in low T, and so βωD/2 will be fairly large since ωD ~ 1/TD where TD ~ room temperature-ish. An integration by parts will extract the largest term,



The remaining integral is small. But a careful analysis gives us the following two terms to largest order:



Feeding this back into the equation, we see that RR(0,0) diverges when,



which works out to:



This is the exact expression we got previously in the MFT file. Note TC is nonzero for any λ > 0. But the algebraic form of TC vs. λ indicates that there is an essential singularity at λ = 0. This means that the Cooper instability can never be captured at finite order in perturbation theory. Instead we must sum an infinite number of terms (as was done in our ladder approximation). And anyway, so we see that at T = Tc the proclivity for electrons to form singlet pairs is infinite, which suggests that the electrons will start en masse to condense into such a state once we hit that temperature, and start to form a new phase.